Consider the family of functions defined by

$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 1}, & x \leq 0, \\ ce^x + d, & x > 0, \end{cases}$$

where $c, d \in \mathbb{R}$. Answer the following:

1. Find all vertical asymptotes.
2. Determine the oblique asymptote using the formulas

$$a = \lim_{x \to \infty} \frac{f(x)}{x}, \quad b = \lim_{x \to \infty} \left(f(x) - ax \right).$$

3. Determine all horizontal asymptotes.
4. Find the values of c and d that make the function continuous and differentiable at $x = 0$.
5. Find all inflection points.
6. Identify all local maxima and minima.
7. Determine the intervals of concavity and convexity.

Step 1: Vertical asymptote

Denominator $x - 1 = 0 \implies x = 1$

Step 2: Oblique asymptote

Use formulas $a = \lim_{x\to\infty} f/x, b = \lim_{x\to\infty}(f - ax)$:

$$a = \lim_{x\to\infty} \frac{x^2 - 3x + 2}{x(x-1)} = \lim_{x\to\infty} \frac{1 - 3/x + 2/x^2}{1 - 1/x} = 1$$

$$b = \lim_{x\to\infty} \left(\frac{x^2 - 3x + 2}{x - 1} - x\right) = \lim_{x\to\infty} \frac{x^2 - 3x + 2 - x(x-1)}{x - 1} = \lim_{x\to\infty} \frac{-2x + 2}{x - 1} = -2$$

Oblique asymptote: $y = x - 2$

Step 3: Horizontal asymptote

Second piece: $ce^x + d \to \infty$ → none

Step 4: Continuity and differentiability at $x = 0$

Left-hand limit: $f(0^-) = \frac{0-0+2}{0-1} = -2$

Right-hand side: $f(0^+) = c + d$ → set equal for continuity: $c + d = -2$

Derivative of first piece:

$$f_1'(x) = \frac{(2x-3)(x-1) - (x^2 - 3x + 2)}{(x-1)^2} = \frac{x^2 - 2x - 1}{(x-1)^2}$$

$f_1'(0) = \frac{0-0-1}{1} = -1$

Derivative of second piece: $f_2'(0) = c$ → set equal: $c = -1$ → then $d = -1$

So continuity and differentiability: $c = -1, d = -1$

Step 5: Inflection points

Second derivative:

$$f_1''(x) = \frac{d}{dx}\frac{x^2 - 2x - 1}{(x-1)^2} = \frac{2(x-1)^2 - (x^2 - 2x - 1) * 2(x-1)}{(x-1)^4} = \frac{2(x-1) - 2(x^2 - 2x - 1)}{(x-1)^3} = \frac{-2x^2 + 6x}{(x-1)^3}$$

Set numerator $= 0 \implies -2x^2 + 6x = 0 \implies x(-2x + 6) = 0 \implies x = 0, 3$ → domain $x \le 0$ → inflection point at $x = 0$