

3D Geometry – Comprehensive Multi-Part Problem

Instructions: Solve the following multi-part problem on lines and planes in 3D. Show all your work.

A line and two planes are given in 3D space:

- Line L (parametric form):

$$\mathbf{r}(t) = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, t \in \mathbb{R}$$

- Plane Π_1 (Cartesian form):

$$x - y + 2z - 3 = 0$$

- Plane Π_2 (vector form):

$$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, s, t \in \mathbb{R}$$

Questions

(a) Line-Plane Intersection

Find the point where the line L intersects the plane Π_1 .

(b) Line-Plane Angle

Find the acute angle between line L and plane Π_1 .

(c) Plane-Plane Intersection

Write the equation of the line of intersection of planes Π_1 and Π_2 in **parametric form**.

(d) Distance from Point to Plane

Find the distance from the point on L corresponding to $t = 1$ to plane Π_2 .

(e) Angle Between Line and Intersection Line

Find the angle between line L and the line of intersection obtained in part (c).